Geometry: 10.4-10.7 Notes NAME____________________

10.4 Inscribed Angles in Polygons____________________________ Date:_________________

Define Vocabulary:

inscribed angle –

intercepted arc

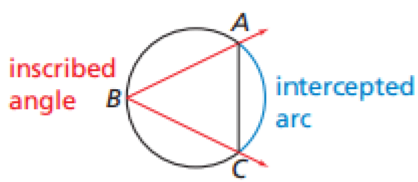
subtend

inscribed polygon

circumscribed circle

Inscribed Angle and Intercepted Arc

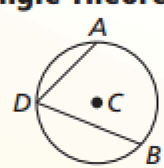
An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. An arc that lies between two lines, rays, or segments is called an **intercepted arc**. If the endpoints of a chord or arc lie on the sides of an inscribed angle, then the chord or arc is said to **subtend** the angle.



$\angle B$ intercepts $\overset{\frown}{AC}$.
$\overset{\frown}{AC}$ subtends $\angle B$.
$\overline{AC}$ subtends $\angle B$.

Theorem 10.10 Measure of an Inscribed Angle Theorem

The measure of an inscribed angle is one-half the measure of its intercepted arc.



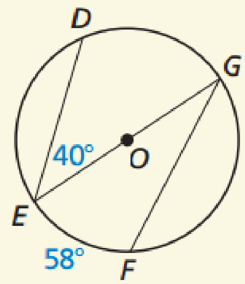
$m\angle ADB = \frac{1}{2} m\overset{\frown}{AB}$

Proof Ex. 37, p. 560

Examples: Using Inscribed Angles

WE DO

Find the indicated measure.

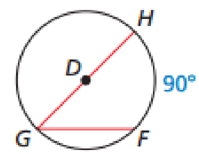


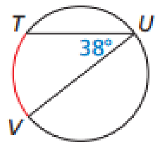
a. $m\widehat{DG}$

b. $m\angle G$

YOU DO

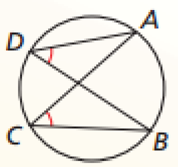
Find the measure of the red arc or angle.

1. 

2. 

Theorem 10.11 Inscribed Angles of a Circle Theorem

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.



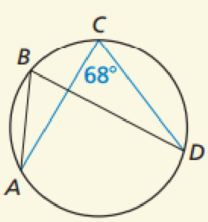
$\angle ADB \cong \angle ACB$

Proof Ex. 38, p. 560

Examples: Finding the Measure of an Angle

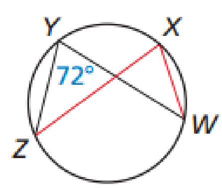
WE DO

Given $m\angle C = 68°$, find $m\angle B$.



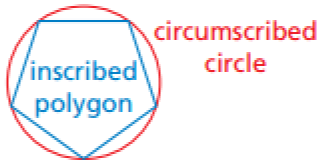
YOU DO

Find the measure of the red arc or angle.



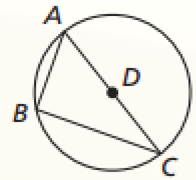
Inscribed Polygon

A polygon is an **inscribed polygon** when all its vertices lie on a circle. The circle that contains the vertices is a **circumscribed circle**.



Theorem 10.12 Inscribed Right Triangle Theorem

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

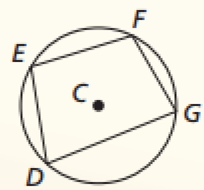


$m\angle ABC = 90°$ if and only if $\overline{AC}$ is a diameter of the circle.

Proof Ex. 39, p. 560

Theorem 10.13 Inscribed Quadrilateral Theorem

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.



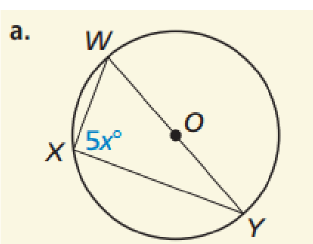
D, E, F, and G lie on $\odot C$ if and only if $m\angle D + m\angle F = m\angle E + m\angle G = 180°$.

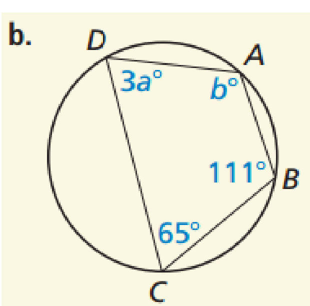
Proof Ex. 40, p. 560; *BigIdeasMath.com*

Examples: Using Inscribed Polygons

<u>WE DO</u>

Find the value of each variable.

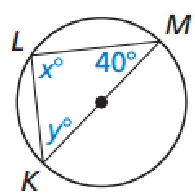




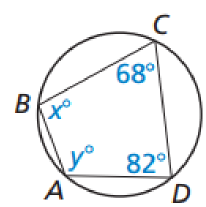
<u>YOU DO</u>

Find the value of each variable.

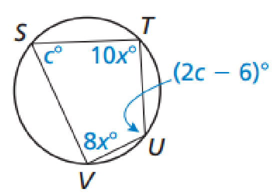
1.



2.



3.



Assignment	

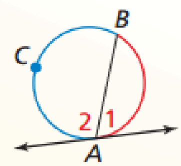
10.5 Angle Relationships in circles_______________ _____________________________ Date:______________

Define Vocabulary:

circumscribed angle

Theorem 10.14 Tangent and Intersected Chord Theorem

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc.



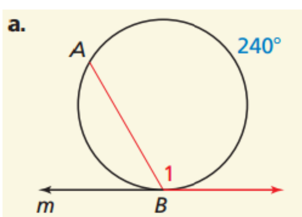
Proof Ex. 33, p. 568

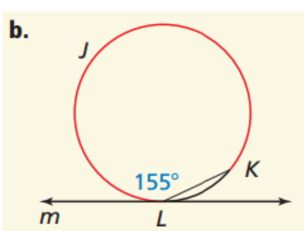
$m\angle 1 = \frac{1}{2}m\overset{\frown}{AB}$ $m\angle 2 = \frac{1}{2}m\overset{\frown}{BCA}$

Examples: Finding Angle and Arc Measures

WE DO

Line m is tangent to the circle. Find the measure of the red angle or arc.

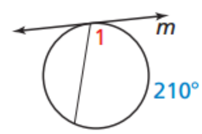




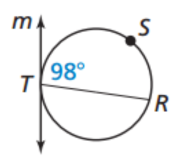
YOU DO

Line m is tangent to the circle. Find the indicated measure.

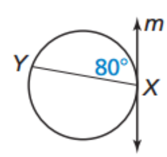
1. $m\angle 1$



2. $m\overset{\frown}{RST}$

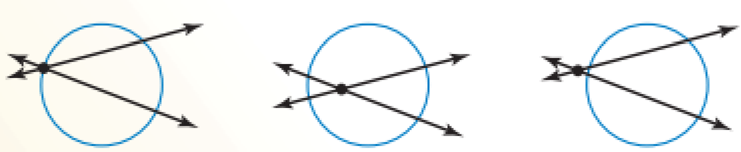


3. $m\overset{\frown}{XY}$



Intersecting Lines and Circles

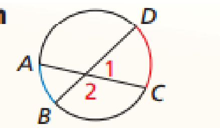
If two nonparallel lines intersect a circle, there are three places where the lines can intersect.



on the circle inside the circle outside the circle

Theorem 10.15 Angles Inside the Circle Theorem

If two chords intersect *inside* a circle, then the measure of each angle is one-half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.



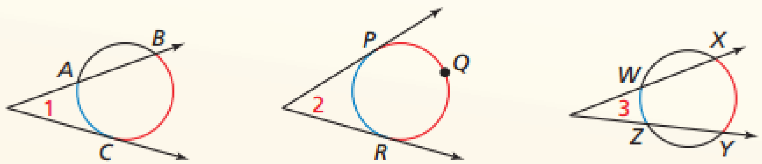
$m\angle 1 = \frac{1}{2}(m\widehat{DC} + m\widehat{AB})$,

$m\angle 2 = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$

Proof Ex. 35, p. 568

Theorem 10.16 Angles Outside the Circle Theorem

If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one-half the *difference* of the measures of the intercepted arcs.

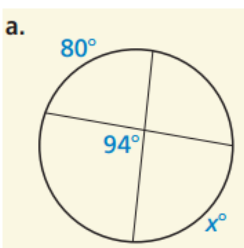


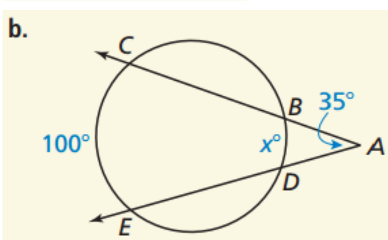
$m\angle 1 = \frac{1}{2}(m\widehat{BC} - m\widehat{AC})$ $m\angle 2 = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$ $m\angle 3 = \frac{1}{2}(m\widehat{XY} - m\widehat{WZ})$

Examples: Finding an Angle Measure

<u>WE DO</u>

Find the value of x.

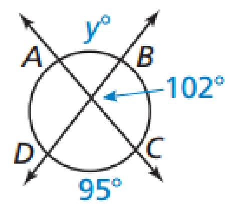




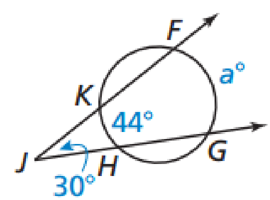
<u>YOU DO</u>

Find the value of the variable.

1.

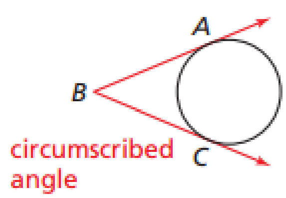


2.



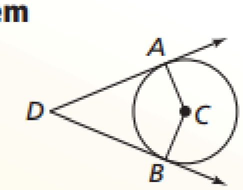
Circumscribed Angle

A **circumscribed angle** is an angle whose sides are tangent to a circle.



Theorem 10.17 Circumscribed Angle Theorem

The measure of a circumscribed angle is equal to 180° minus the measure of the central angle that intercepts the same arc.



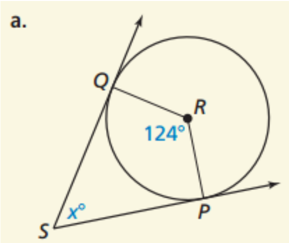
Proof Ex. 38, p. 568

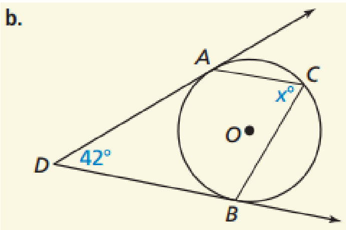
$m\angle ADB = 180° - m\angle ACB$

Examples: Finding Angle Measures

WE DO

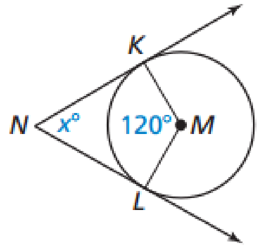
Find the value of x.



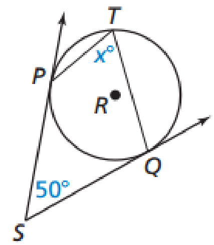


YOU DO

1.



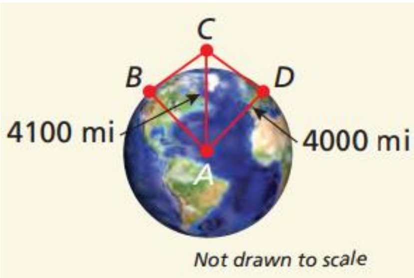
2.



Examples: Modeling with Mathematics

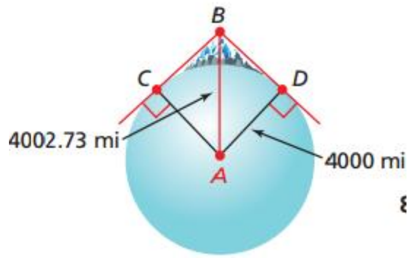
WE DO

A flash occurs 100 miles above Earth at point C. Find the measure of BD, the portion of Earth from which the flash is visible.



YOU DO

You are on top of Mount Rainier on a clear day. You are about 2.73 miles above sea level at point B. Find m CD, which represents the part of Earth that you can see.



Assignment	

Define Vocabulary:

segments of a chord

tangent segment

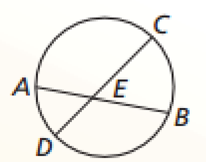
secant segment

external segment

Theorem 10.18 Segments of Chords Theorem

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

Proof Ex. 19, p. 574

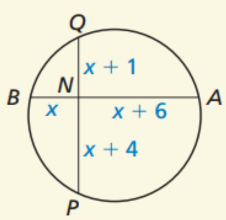


$EA \cdot EB = EC \cdot ED$

Examples: Using Segments of Chords

WE DO

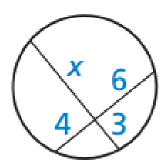
Find AB and PQ.



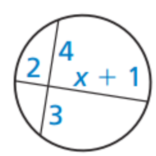
YOU DO

Find the value of x.

1.

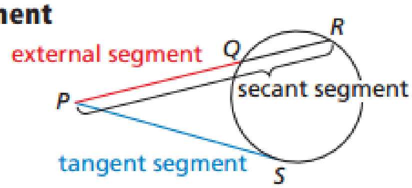


2.



Tangent Segment and Secant Segment

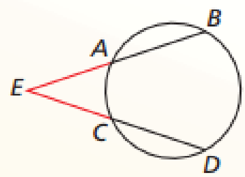
A **tangent segment** is a segment that is tangent to a circle at an endpoint. A **secant segment** is a segment that contains a chord of a circle and has exactly one endpoint outside the circle. The part of a secant segment that is outside the circle is called an **external segment**.



$\overline{PS}$ is a tangent segment.
$\overline{PR}$ is a secant segment.
$\overline{PQ}$ is the external segment of $\overline{PR}$.

Theorem 10.19 Segments of Secants Theorem

If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.



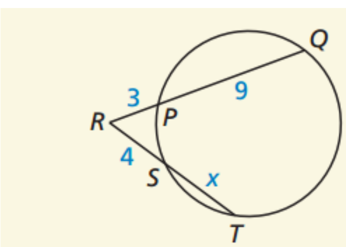
Proof Ex. 20, p. 574

$EA \cdot EB = EC \cdot ED$

Examples: Using Segments of Secants

WE DO

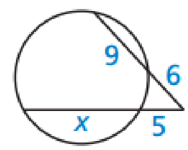
Find the value of x.



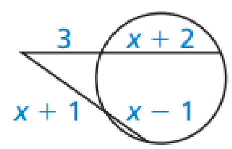
YOU DO

Find the value of x.

1.

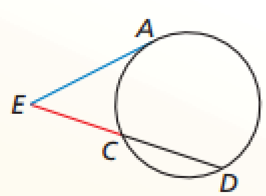


2.



Theorem 10.20 Segments of Secants and Tangents Theorem

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.



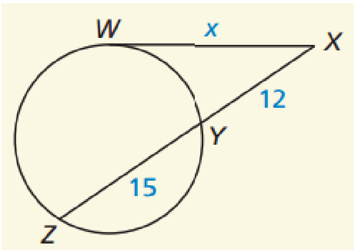
Proof Exs. 21 and 22, p. 574

$EA^2 = EC \cdot ED$

Examples: Using Segments of Secants and Tangents

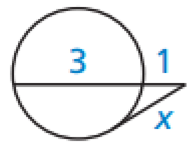
WE DO

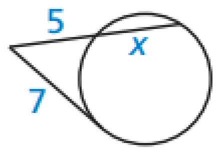
Find *WX*.



YOU DO

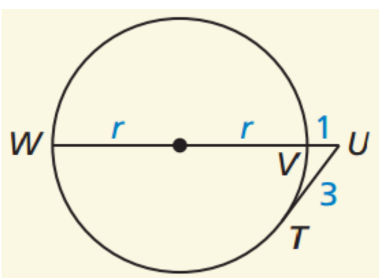
Find the value of x.





Examples: Find the radius of the circle.

WE DO



Assignment	

<u>10.7 Circles in the coordinate plane</u>______________________________________**<u>Date:</u>**____________

Define Vocabulary:

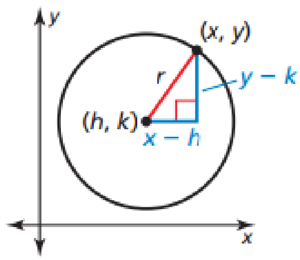
standard equation of a circle

Standard Equation of a Circle

Let (x, y) represent any point on a circle with center (h, k) and radius r. By the Pythagorean Theorem (Theorem 9.1),

$$(x - h)^2 + (y - k)^2 = r^2.$$

This is the **standard equation of a circle** with center (h, k) and radius r.

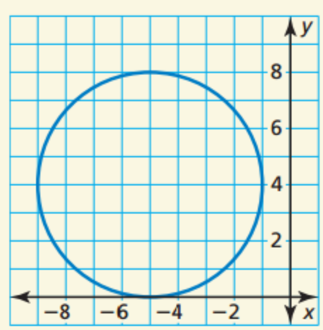


Examples: Writing the Standard Equation of a Circle

<u>WE DO</u>

Write the standard equation of each circle.

a.



b. a circle with center at the origin and radius 3.5.

<u>YOU DO</u>

Write the standard equation of the circle with the given center and radius.

1. center: (0, 0), radius: 2.5

2. center: (−2, 5), radius: 7

Examples: Writing the Standard Equation of a Circle

<u>WE DO</u>

The point (4, 1) is on a circle with center (1, 4). Write the standard equation of the circle.

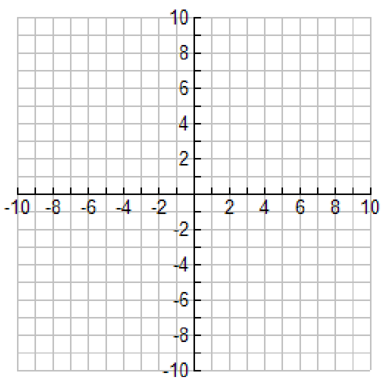
<u>YOU DO</u>

The point (3, 4) is on a circle with center (1, 4). Write the standard equation of the circle.

Examples: Graphing a Circle

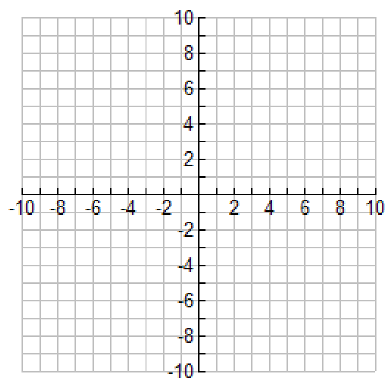
<u>WE DO</u>

The equation of a circle is $x^2 + y^2 - 2x + 6y - 6 = 0$. Find the center and the radius of the circle. Then graph the circle.



<u>YOU DO</u>

The equation of a circle is $x^2 + y^2 - 8x + 6y + 9 = 0$. Find the center and the radius of the circle. Then graph the circle.



Assignment	